Background

- Four Primary Goals:
  - Develop a quantum computer with a sufficiently large number of qubits to solve a challenging calculation.
  - Ensure that every qubit interacts with all other qubits in the system, critical for solving fundamental problems in physics. *(long-range, nonlocal interactions)*
  - Integrate software, algorithms, devices and systems engineering.
  - **Co-designing** the quantum computer with experimentalists, theorists, engineers and computer scientists.

![Graphic representations of the two systems. (A) the superconducting qubits connected by microwave resonators (credit: IBM Research). (B) The linear chain of trapped ions connected by laser-mediated interactions. (A and B, Insets) Qubit connectivity graphs: (A) star shaped and (B) fully connected.](image)
The speed of running a quantum algorithm on a quantum computer is typically limited by two-qubit entangling gates, as single qubit gates in most platforms are 10-1000x faster than two-qubit gates.

For a fully connected quantum computer, we can use the connectivities among all qubits to increase the speed of entanglement creation dramatically, via engineered constructive quantum interference.

Example 1: Remote quantum gate time can be reduced from $O(N)$ to $O(1/\sqrt{N})$.

Example 2: Creating a topologically ordered (long-range entangled) state need just $O(\log N)$ instead of $O(N)$.

**Main Question:** What is the speed limit of entangling two (or more) qubits in a fully connected quantum computer?

\[ + = ? \]

Let us consider two limiting scenarios for our main question:

- **Number of qubits is small**: Given the form of the achievable Hamiltonian, what is the fastest protocol to achieve a given two-qubit unitary?

  \[
  H(t) = J \sum_{i,j=1}^{3} \sigma^z_i \sigma^z_j + \sum_{i=1}^{3} B_i(t) \sigma^x_i \quad \text{min } T(U_{\text{CNOT}_{1,3}}) \quad \text{subject to} \quad \sigma^z_i \sigma^z_j \geq \epsilon
  \]

- **Number of qubits is large**: Given the decay of interactions in distance, what is the **scaling** of any remote two-qubit gate time with distance or the total number of qubits?

  \[
  H(t) = \sum_{i<j} J_{ij}(t) h_{ij} + \sum_i B_i(t) h_{ii} \quad \text{min } T(\|[A_i(t), A_j]\| \geq \epsilon) \quad \text{subject to} \quad \|h_{ij}\| = 1, |J_{ij}(t)| \leq 1/r_{ij}^\alpha
  \]
Entanglement speed limit for two qubits

❖ For simplicity, let’s assume we have two qubits with a Ising interaction of strength unity. We further assume arbitrary single qubit gates can be performed in negligible time.

❖ There is an analytical method for finding the fastest protocol to generate any given two-qubit gate, via the following decomposition (known as Cartan decomposition):

\[ U = (u_1 \otimes u_2)e^{-iT\sigma_1^z\sigma_2^z}(v_1 \otimes v_2) \]

❖ For CNOT gate: \( T_{\text{min}} = \pi/4 \).

❖ For XY gate: \( T_{\text{min}} = \pi/2 \).

❖ For SWAP gate: \( T_{\text{min}} = 3\pi/4 \).

Three qubits

- There’s no proven speed limit for generating a given two-qubit (or 3-qubit) unitary for a system of three interacting qubits.

- We have proven speed limit in some constrained scenarios:
  - In particular, if the Hamiltonian can be mapped to free particles. We can prove that the XY gate for two of the three qubits takes $T_{min} = \frac{\pi}{\sqrt{6}}$, a speedup compared to the two-qubit case ($T_{min} = \frac{\pi}{2}$).
  - However, removing the free-particle constraint allows one to achieve a faster gate. For example, we found a protocol for XY gate that takes time $T = \frac{\pi}{\sqrt{8}}$ by using simply Ising type interactions.
Experimental demonstration

- **Experimental setup at NIST**: Capacitively coupled transmon qubits with Ising type interaction. (See Poster by Joel Howard tomorrow for details)

- **Complication**: Single qubit gate time not negligible compared to the two-qubit interaction strength.

- Need new theoretical proofs for the speed limit as a function of both single qubit drive strength and two-qubit interaction strength.

- Numerical optimization can be performed to guess the speed limit.
Many qubits

- For any free particle Hamiltonian of the form:
  \[ H(t) = \sum_{i,j \in \Lambda} (J_{ij}(t)c_i^\dagger c_j + \text{h.c.}) + \sum_{i \in \Lambda} B_i(t)c_i^\dagger c_i. \]

- We have proven that any two-qubit entangling gate has to take a minimum time of
  \[ T_{\text{min}}(U_{ij}) \geq \frac{1}{\sqrt{\min\left(\sum_k J_{ik}^2, \sum_k J_{kj}^2\right)}} \sim 1/\sqrt{N}. \]

- Importantly, this minimum time scaling is achievable using practical protocols.

The goal of state transfer is to move sites. The strengths of the hopping terms are bounded by a power-law that virtually vary with the distances between sites.

Here, \( H(t) \) is initially in the state \( \langle 0 \rangle_X | 0 \rangle^{\otimes N-2} | 0 \rangle_Y \). While that protocol \( T = \pi L^\alpha / \sqrt{N-2} \), the above protocol requires a time that increases with \( t \), allowing interactions may increase the rate of information transfer.

We label the sites that are not \( \langle 0 \rangle_X \) \( \langle 0 \rangle^{\otimes N-2} \) \( | 0 \rangle_Y \). Further evolving the state by \( e^{-iH_1T/2} \langle 0 \rangle^{\otimes N} + b | 0 \rangle_X | W \rangle | 0 \rangle_Y \).

\[ H(t) = \begin{cases} H_1 = \frac{1}{L^\alpha} \sum_{i=1}^{N-2} c_X^\dagger c_i + \text{h.c.} & 0 < t < T/2, \\ H_2 = \frac{1}{L^\alpha} \sum_{i=1}^{N-2} c_i^\dagger c_Y + \text{h.c.} & T/2 < t < T, \end{cases} \]

But can we do better with Hamiltonians that cannot be mapped to free particles?
General Hamiltonians

- Consider the most general two-body interacting Hamiltonian:

\[ H(t) = \sum_{i,j} J_{ij}(t) h_{ij} + \sum_i B_i(t) h_i \]

- We proved that the minimal time to entangle any two qubits is given by (using the techniques of deriving Lieb-Robinson bound)

\[ T_{\text{min}} = O(\log(N)/N) \]

- And the minimal time to entangle any two parts of the system is given by

\[ T'_{\text{min}} = O(1/N) \]

- However, unlike the free particle Hamiltonians, no known Hamiltonian can actually realize either of the minimal entangling time scaling above. It’s highly likely that these scalings are loose and can be improved further.

Quantum Information Scrambling

- The minimal time to entangle two parts of a quantum many-body system directly bounds the time it takes for a quantum system to **scramble**: 
  - A quantum system is said to be scrambled if any information initially contained in a subsystem can no longer be recovered from measurement on that subsystem alone.
  - Scrambling is believed to be happening when matter disappears in a black hole (the information attached to that matter get chaotically mixed with all other matter).
  - Black holes are conjectured to be the fastest information scramblers in the universe. It will scramble in a time logarithmic in the number of degrees of freedom.
  - Many proposed quantum models of black holes include highly nonlocal interactions. A fully-connected quantum computer can thus simulate a mini blackhole. [K. Landsman et al, Nature 2019]
  - We would have proven the long-standing fast scrambling conjecture if we can tighten our entangling speed limit bound!
Many open questions:

- Is there an analytical method for finding the fastest gate sequence for achieving a specific two-qubit entangling gate for 3 or more qubits? If not, can we find an efficient numerical method at least?
- How about 3-qubit entangling gates such as the Toffoli gate commonly used in quantum circuits?
- How should we account for gate errors in these optimal circuit designs?
- Is there actually a scaling difference between free particle Hamiltonians and their counterparts for the entangling speed limit?
- Towards the fast scrambling conjecture: Can we close the gap between the two-qubit and two-subsystem entangling speed limits?
Acknowledgement

❖ **Students at Mines**
  ❖ Joel Howard
  ❖ Casey Haack, Casey Jameson
  ❖ Kyle Clark, Daniel Moore, Ryan Hooker.

❖ **Experimental collaborators:**
  ❖ Meenakshi Singh (Mines)
  ❖ Junling Long (NIST)
  ❖ David Pappas (NIST)

❖ **Funding Support** from NSF RAISE-TAQS program.